Date: 25.03.2014
Teacher: Ezgi Çallı
Number of Students:
Grade Level: 10 (L10-2)
Time Frame: 80 minutes

Mathematics Learning Plan

1. Goal(s)

- To develop an understanding of the binomial theorem, Pascal's triangle and their interrelations.

2A. Specific Objectives

- Students will recognize Blaise Pascal (1623-1662).
- Students will propose different number patterns in the Pascal's triangle.
- Students will use the Pascal's Triangle to find the binomial coefficients in a binomial expansion.
- Students will express the binomial theorem by using the sigma notation.
- Students will determine the coefficient of any term in a binomial expansion.
- Students will be able to use technology (TI-84) to find combinations (nCr) and to write the nth line of the Pascal's triangle as a sequence (seq( )).

2B. Ministry of National Education (MoNE) Objectives

- 10.1.1.4. Kombinasyon kavramının aşağıdaki temel özellikleri incelenir.

$$C\binom{n}{r} = C\binom{n}{n-r}$$

$$C\binom{n}{0} + C\binom{n}{1} + C\binom{n}{2} + \cdots C\binom{n}{n} = 2^n$$

- 10.1.1.5. Pascal özdeşliğini gösterir ve Pascal üçgenini oluşturur.
- 10.1.1.6. Binom teoremini açıklar ve açılımdaki katsayıları Pascal üçgeni ile ilişkilendirir.

2C. IB Standards:

- The binomial theorem: Expansion of $(a + b)^n$, $n \in \mathbb{N}$.
- The ability to find $\binom{n}{r}$ by using both the formula and technology is expected.

3. Rationale

- Pascal's triangle and the binomial theorem set a foundation for many big concepts such as bioinformatics (for e.g. cell division), statistics, game theory (economics), actuarial science, and so forth. Especially those students, who wish to study computer science, will use this kind of number patterns very frequently in their profession.

4. Materials

SMART board

SMART Notebook

Board markers

TI calculators

Worksheets

Lap tops

5. Resources

- Advanced Mathematics by Richard. G. Brown, 2011
- A link for the proof of binomial theorem by induction: http://www.math.hmc.edu/calculus/tutorials/binomial_thm/induction.html
- Pascal documentary: https://www.youtube.com/watch?v=wN_gGX96im8
- Patterns in Pascal's triangle: http://www.cut-the-knot.org/arithmetic/combinatorics/PascalTriangleProperties.shtml

6. Getting Ready for the Lesson (Preparation Information)

- Make sure the projector works
- Go to the class early and arrange the SMART board settings
- 2 worksheets should be printed before the lesson.

7. Prior Background Knowledge (Prerequisite Skills)

- Students should be able to expand powers of binomials algebraically.
- Students should have learned permutations and combinations.

Lesson Procedures

Transition: "Good morning. We met with some of you last week, Let me introduce myself. I am Ezgi Çallı from Bilkent University Graduate School of Education. (Teacher writes her name/surname and e-mail address for further contact) I am from Ankara. I have my bachelor's degree from METU mathematics department, and I have a master's degree from the Industrial Engineering department of the same university. I want

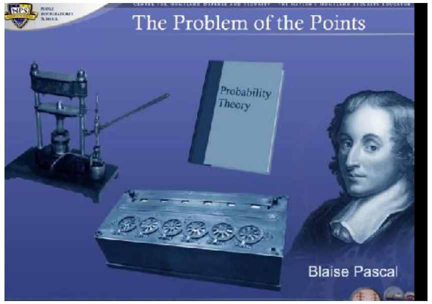
you to write your names on a piece of paper and put on your desks please. Today we're going to spend the 80 minutes with you.
Before starting the lesson, I would like to remind you a few points about the flow of the lesson. I expect you to raise hands before taking the floor. OK?
The second thing I would like to remind is that you'll need your computers at the last 20 minutes of the lesson for a short quiz on google forms. Therefore, if it takes a long time for your computer to be ready, then start them and wait in the standby mode. Does anyone need it?

8A. Engage 10 minutes

The teacher shows a picture of Blaise Pascal and asks if anyone knows him. Then she asks: "Who is the father of calculator"?

He was a French mathematician, physicist, philosopher who lived in the 17^{th} century.
When he was 12, he showed that the sum of angles in a triangle equals to two right angles.
He invented the first mechanical calculator, called Pascaline.
Actually, Pascal's triangle was discovered in the 13^{th} century. However, it is known with his name. We'll talk about why this might have happened at the end of the lesson.



Transition: Let's have a closer look at this Pascal's triangle. (The teacher distributes the worksheet) First of all, let us start with expanding the given powers of binomials. Expand the given polynomials and write the coefficients of the terms for

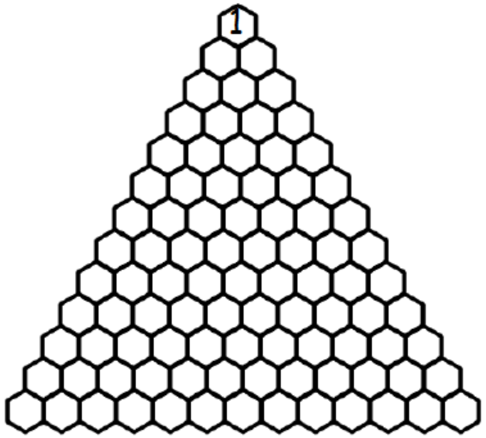
B. Explore 15 minutes

$(a+b)^1$,$(a+b)^2$, $(a+b)^3$, $(a+b)^4$ and so on.

Ask the students to simplify their answers by collecting the like terms.

Some students might show their algebraic manipulations on the board.

Then, ask the students to write the coefficients on the cells given on the paper.



Ask the students if they observe any pattern? Wait for them to discover. Remind that this **individual work.**

Do they observe any other relationship than the binomial coefficients? (Fibonnaci, triangular, even numbers, powers of 11, etc.)

They can present at the board.

Transition: After investigating the Pascal triangle deeply, let's go back to binomial expansion.

C. Explain (20 minutes)

You've found the coefficients in the 5th power, or 6th power, etc. of the binomial expansion. But! What if the power is a very large number such as 45? Are you going to write the 45th line of the Pascal's triangle?

- How can we write the general term of a binomial expansion?
- The teacher expects the students to discover how this triangle is related with combinations.
- Provide the binomial theorem and the general term in the expansion of$(a+b)^n$.
- Ask how many terms are there in this expansion. What is the sum of powers?

- How can we show by using the sigma notation?
- Why is it related with combinations?
- Show the following relation by referring to sets and with a real life example such as the number of non-defective items in a set of 10 items or number of heads when tossing a coin 10 times.
  $C\binom{n}{0} + C\binom{n}{1} + C\binom{n}{2} + \cdots C\binom{n}{n} = 2^n$

  AT LEAST / AT MOST QUESTION (Probability extension)

Transition: Let us solve some questions by using what we have learnt so far. These are generally national university entrance exam type questions.

D. Extend (15 minutes)

- 2 questions solved at the board.

  Example 1. Give the first four terms in the expansion of $(x-2y)^{10}$in simplified form.

  Solution:

  $$x^{10} - 20x^2y + 180x^8y^2 - 960x^7y^3 = (x-2y)^{10}$$

  Example 2. In the expansion of $(3+y^2)^9$, find the term containing y^{10}.

  Solution:$\binom{9}{5}.3^4.y^{10} = 10206y^{10}$

  TI ACTIVITY

- The teacher encourages students to apply or extend the concepts and skills in new situations. She reminds students of alternative explanations.
- Students are encouraged to use their graphing calculators to find the combinations and the binomial coefficients.
- DAILY ANNOUNCEMENTS
- Then, a worksheet is given to the students if there is extra time.

Transition: The teacher writes the quiz link on the board.

E. Evaluate (20 minutes)

- QUIZ- Google Form Link: http://goo.gl/e8x35o Answers: 1.C 2.D 3.B

  9. Closure & Relevance for Future Learning (5 minutes)

- The students will be asked to write one or two sentence feedback about the lesson.

- Refer to the engagement.

10. Specific Key Questions:
    - How many terms are there in the expansion of $(a + b)^n$?
    - What is the sum of degrees in each term?
    - The powers on “a” begin with …. and decrease to ….?
    - The powers on “b” begin with …. and increase to ….? (bloom tax)
    - Is there symmetry in the coefficients?
    - Objectives: knowledge, comprehension, application, analysis, synthesis, and evaluation.