MTE 503-1 Computer Technology in Mathematics Education

TI Worksheet #2

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Class level: 11

Objectives:

IB-DP MATHEMATICAL STUDIES SL

4.4. The χ^2 test for independence: formulation of null and alternative hypotheses; significance levels; contingency tables; expected frequencies; degrees of freedom; p-values.

MoNE

(Extension to) TD.11.5.1. Gerçek hayat durumlarıyla ilgili bir istatistik problemini çözmek için verileri toplar, düzenler, temsil eder ve yorumlar.

Materials: TI Calculator

Time: 45 minutes

Resources:

- Hines, W. W., Montgomery, D. C., & Borror, D. M. G. C. M. (2003). *Probability and statistics in engineering*. John Wiley & Sons.
- INVCHI2, by Stan Brown, Tompkins Cortland Community College, Dryden NY. http://www.tc3.edu/instruct/sbrown/ti83/invchisq.htm

The χ^2 Test of Independence

A company has to choose among three pension plans. Management wishes to know whether the preference for pension plan is independent of job classification. The opinions of a random sample of 500 employees are shown in the following table. Test at a 5 % significant level whether there is an association between job classification and preference for pension plans.

	PENSION PLAN 1 (P1)	PENSION PLAN 2 (P2)	PENSION PLAN 3 (P3)	TOTALS
Salaried workers (SW)	160	140	40	**340**
Hourly workers (HW)	40	60	60	**160**
TOTALS	**200**	**200**	**100**	**500**

STEP 1: Understand the Problem

What do we want to know? We want to know whether we can or cannot conclude that the row and column methods of our classification are independent. So,
What is our null hypothesis?

H_0:……

What is our alternative hypothesis?

H_1:……

STEP 2: Formulate the Problem

(Note: The table given to us, which includes the observed frequencies, is called a (2x3) *contingency table*.)

To test our hypothesis, we calculate the values we would expect to get, if the variables were independent. If ***job classification*** and ***preferences for pension plans*** were independent events, then

$$P(SW \cap P_1) = P(salaried\ worker) \times P\ (pension\ 1) = \frac{340}{500} \times \frac{200}{500} = \frac{34}{125}$$

Therefore, in a sample of 500 workers, we would expect $500 \times \frac{34}{125} = 136$ of them to be salaried worker **and** prefers pension plan-1. Similarly, perform the other calculations and complete the expected frequency table.

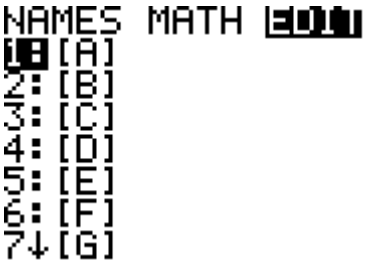
	PENSION PLAN 1 (P1)	PENSION PLAN 2 (P2)	PENSION PLAN 3 (P3)	TOTALS
Salaried workers (SW)	136			**340**
Hourly workers (HW)				**160**
TOTALS	**200**	**200**	**100**	**500**

The $\boldsymbol{\chi^2}$ test is used to investigate the difference between the *observed* values and the *expected* values. ($f_o : observed\ frequency, f_e : : expected\ frequency$)

$$\chi_0^2 = \sum \frac{(f_o - f_e)^2}{f_e}$$

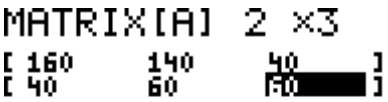
STEP 3: Solve the Problem

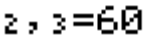
You can use technology to calculateχ_0^2. First of all, enter the contingency table values into a matrix. Press 2^{ND}, then x^{-1} (i.e. MATRIX). Choose the third tab, EDIT. Choose A.



ENTER.

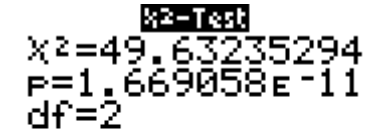
Now enter the number of rows and columns that you have. ENTER. Then, fill in the matrix.





Once the matrix is filled out, press STAT. Scroll twice to the right and choose the third tab, TESTS. In order to find the χ^2 test, scroll down until C. (For a short cut, press ALPHA and PRGM, i.e. C)

Press Calculate. Your calculator is going to find the expected values, and store in Matrix B.



You've found the χ^2value and the p-value. As you can observe:

The degree of freedom = the number of values which are free to vary = 2.

$(df = (2-1) \times (3-1) = 2)$

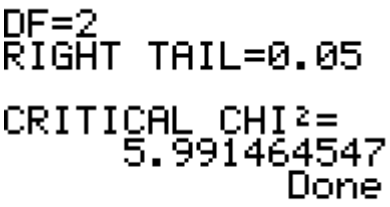
The *significance level* indicates the minimum acceptable probability that the variables are independent. It is given in the question that significance level is 0.05. You can find *the critical* χ^2 *value,* above which you will reject your null hypothesis, by using the degree of freedom and the significance level. One way to find the critical value is to use a χ^2 critical value table. You can find this table easily on the internet or at the appendices of your books.

Alternatively, you can use technology to find this value. There is a nice program called INVCHI2 for TI83/84, which you can use to find the critical value. Send this program to your device. Now, press PRGM and choose INVCHI2.

ENTER.

It asks you the degree of freedom (DF) and significance level (Right Tail).

ENTER. There you go.



Since$\chi^2_{0.05,2} = 5.99$, we reject the hypothesis of independence and we conclude that the pension plan preferences **are not independent** of job classifications.

STEP 4: Look Back

a) Discuss if your conclusion in step 3 is makes sense to you considering real life.

b) You could also use your p-value to make your decision. If your p-value is less than the significance level, then you could reject your null hypothesis. By using this information, find further evidence for your conclusion in step 3.

EXERCISES

Solve the following questions on your own by using your TI calculators.

1) A study is being made of the failures of an electronic component. There are four types of failures possible and two mounting positions for the device. The following data have been taken. Would you conclude that the type of failure is independent of the mounting position?

	Failure Type A	Failure Type B	Failure Type C	Failure Type D	TOTALS
Mounting position 1	22	46	18	9	
Mounting position 2	4	17	6	12	
TOTALS					

2) A company operates four machines for three shifts each day. From production records, the following data on the number of breakdowns are collected. Test the hypothesis that the breakdowns are independent of the shift.

	Machine A	Machine B	Machine C	Machine D	TOTALS
Shift 1	41	20	12	16	
Shift 2	31	11	9	14	
Shift 3	15	17	16	10	
TOTALS					

REFLECTION

Engineers deal with collecting and analyzing numerical data which belongs to the realm of statistics. They observe the performance and trends in various products or processes. Statistics especially plays an important role in the quality assurance of any product or service.

Some examples to this could be estimating the downtimes of a machine, analyzing data about manpower, predicting the reliability of an electronic component, letting a computer make better predictions through machine learning. Therefore, I preferred to choose the examples from engineering concepts in order to link the TI worksheet to STEM education.

In this worksheet, I aimed to encourage high school students to use technology. I chose the chi square independence test, since the statistical concepts such as *hypothesis*, *independence* or *significance* would push the students' conceptual thinking boundaries. Although it is not directly included in the MoNE curriculum, it is clearly available in IB-DP Mathematical Studies program. As it is expected in the curriculum guide of MoNE (2013), I also tried to encourage those students who have limited ability to numerical operations by using TI calculators. While preparing this worksheet, I made use of TI Connect to get the screen captures. I also utilized from a program called INVCHI2 to calculate the critical chi-square value.